

Gauss-Bonnet assisted braneworld inflation in light of BICEP2 and Planck data

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Motivated by the idea that quantum gravity corrections usually suppress the power of the scalar primordial spectrum (E-mode) more than the power of the tensor primordial spectrum (B-mode), in this paper we construct a concrete gravitational theory in five-dimensions for which $V(\phi) \propto \phi^n$ -type inflation ($n \geq 1$) generates an appropriate tensor-to-scalar ratio that may be compatible with the BICEP2 and Planck data together. The true nature of gravity is five-dimensional and described by the action $S = \int d^5x \sqrt{|g|} M^3 (-6\lambda M^2 + R + \alpha M^{-2} \mathcal{R}^2)$ where M is the five-dimensional Planck mass and $\mathcal{R}^2 = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$ is the Gauss-Bonnet (GB) term. The five-dimensional “bulk” spacetime is anti-de Sitter ($\lambda < 0$) for which inflation ends naturally. The effects of \mathcal{R}^2 term on the magnitudes of scalar and tensor fluctuations and spectral indices are shown to be important at the energy scale of inflation. For GB-assisted $m^2\phi^2$ -inflation, inflationary constraints from BICEP2 and Planck, such as, $n_s \simeq 0.9603 (\pm 0.0073)$, $r = 0.16 (+0.06 - 0.05)$ and $V_*^{1/4} \gtrsim 1.5 \times 10^{16} \text{ GeV}$ are all satisfied for $(-\lambda\alpha) \simeq (3 - 300) \times 10^{-5}$.

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I. INTRODUCTION

The 2013 Planck data constrains the scalar spectral index to $n_s = 0.9603 \pm 0.0073$ [1] – a result compatible with that predicted by primordial cosmic inflation [2]. The BICEP2 (Background Imaging of Cosmic Extragalactic Polarization 2) collaboration [3] reports on the detection of inflationary gravitational waves (B-mode polarization), which confirms yet another prediction of inflation. Such results from Planck and BICEP2 offer a rare opportunity to directly test theoretical models, including inflation. BICEP2 results in particular indicate toward some new physics around the energy scale of inflation, $\rho^{1/4} \sim 1.5 \times 10^{16} \text{ GeV}$, which is in the order of symmetry breaking scale of the grand unified theory. The tensor fluctuations in the cosmic microwave background (CMB) temperatures at large angular scales are larger than those predicted for inflationary models based on Einstein gravity. Specifically, the ratio of tensor-to-scalar perturbations reported by BICEP2 collaboration, $r = 0.19_{-0.05}^{+0.07}$ (or $r = 0.16_{-0.05}^{+0.06}$ after subtracting an estimated foreground), is larger than the bounds $r < 0.13$ and $r < 0.11$ reported by WMAP [4] and Planck [1]. Some part of this discrepancy may be accounted for by postulating that the value of r measured by BICEP2 at $\ell \simeq 60$ corresponds to a smaller field of view of the sky where an inflationary gravitational waves signal would be expected to peak, whereas the value of r measured by Planck at $\ell \simeq 30$ corresponds to a larger field of view of the sky where r gets attenuated. In this paper we identify a concrete gravitational theory in which the value of r gets enhanced due to quantum gravity corrections or higher-curvature terms. The latter suppress the scalar primordial power spectrum (PPS) more than the tensor PPS at high energies (compared to the results in general relativity) for ϕ^n -type potentials ($n \geq 2/3$). This is one of the key results of this paper.

One way to accommodate quantum effects of gravity is to include curvature-squared terms in a gravitational action. Such terms arise from the low energy effective action of string theory and/or as the $1/\mathcal{N}$ corrections in the large \mathcal{N} limit of some gauge theories [5]. The Gauss-Bonnet (GB) combination of curvature invariants, $\mathcal{R}^2 = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$, is of particular relevance in five dimensions, since it represents the unique combination that leads to second-order gravitational field equations and hence to ghost-

free solutions in flat as well as in curved spacetimes [6].

Inflationary constraints from Planck data have been based on general relativity. An exponential potential (also called power law inflation) is *not* favored by Planck data for two reasons; one reason is that the value of r is relatively large, $r \simeq -8(n_s - 1) \simeq 0.32$, which is beyond 2σ confidence level of the Planck data ($r < 0.26$, 95% confidence). The second reason, though not limited to exponential potential, is that inflation would not end without an additional mechanism to stop it. A question of great significance is: What happens if the early universe is better described by the following Einstein-Gauss-Bonnet gravitational action in five dimensions?

$$S = \int_{\mathcal{M}} d^5x \sqrt{|g|} M^3 \left(-6\lambda M^2 + R + \frac{\alpha}{M^2} \mathcal{R}^2 \right) + \int_{\partial\mathcal{M}} d^4x \sqrt{|\tilde{g}|} (\mathcal{L}_m + \mathcal{L}_\phi - \sigma), \quad (1)$$

where M is the five-dimensional Planck mass, \mathcal{L}_m (\mathcal{L}_ϕ) is matter (scalar) Lagrangian and σ is the brane tension (or a cosmological constant in four dimensions). The above action is consistent with *braneworld* realisation [9] of string and M theory, according to which all elementary particles, gauge fields and fundamental scalars live within a four-dimensional (3 dimensions of space and 1 dimension of time) membrane, or “brane,” while the effect of gravity extends along the fifth dimension. The condition $-\lambda\alpha < \mathcal{O}(1/10)$ [6] is required for the stability of classical solutions under perturbations, which also guarantees a suppression of higher powers of curvature tensors. Inflationary constraints provide a more stringent bound, $(-\lambda\alpha) < \mathcal{O}(10^{-3})$. In this paper, we show that the above mentioned model leads to amazingly simple four-dimensional universe that resembles in many ways the one observed by BICEP2 and Planck.

II. MODIFIED FRIEDMANN EQUATIONS

We are interested in cosmological solutions, so we write the 5D metric *ansatz* as

$$ds^2 = -N(t, y)^2 dt^2 + A^2(t, y) d\Omega_{3,\kappa}^2 + B(t, y)^2 dy^2. \quad (2)$$

The use of gauge $N(t, y = 0) = N_0 \equiv 1$ implies $\dot{A} = aH\dot{N}$ (where $a \equiv A_0$ is the scale factor of the Friedmann-Lemaître-

Robertson-Walker universe) and t is the 4D proper time. The set of four-dimensional field equations are given by

$$X \left[1 + \frac{4\alpha H^2}{M^2} \left(1 - \frac{X^2}{3H^2} + \frac{k}{a^2 H^2} \right) \right] = -\frac{(\rho + \sigma)}{6M^3}, \quad (3)$$

$$X \left[1 + \frac{4\alpha H^2}{M^2} \left(1 + \frac{\dot{H}}{H^2} - \frac{XY}{3H^2} \right) \right] + \frac{Y}{2} \left[1 + \frac{4\alpha H^2}{M^2} \left(1 - \frac{X^2}{3H^2} + \frac{k}{a^2 H^2} \right) \right] = \frac{(p - \sigma)}{4M^3}, \quad (4)$$

(see also Refs. [7, 8]) where

$$X \equiv \frac{A'|_{y=0}}{aB_0} = -\sqrt{H^2 + \psi^2 M^2 + \frac{k}{a^2}}, \quad Y \equiv \frac{N'|_{y=0}}{B_0}.$$

$$\psi^2 = \frac{1 - \sqrt{\Delta}}{4\alpha} \quad \text{and} \quad \Delta \equiv 1 + 8\lambda\alpha + \frac{8\alpha}{a^4} \frac{\mathcal{E}}{M^2}.$$

\mathcal{E} is a measure of bulk radiation energy, which is proportional to the mass of a 5D black hole. ψ is a dimensionless measure of bulk curvature. In the case the 5D spacetime is anti-de Sitter ($\lambda < 0$) and the GB coupling constant is positive ($\alpha > 0$), such that $\Delta < 1$, the \mathcal{R}^2 -type corrections would lead to graceful exit from inflation for a number of scalar potentials.

To study inflation, we will ignore the term $\kappa/a^2 H^2$, which is justified from the viewpoint that inflation would stretch any initial curvature of the universe to near flatness*. The Friedmann equation (3) can be written as

$$H^2 = \frac{M^2 \psi^2}{\beta} [(1 - \beta) \cosh \varphi - 1], \quad (5)$$

$$\varphi \equiv \frac{2}{3} \sinh^{-1} \left(\sqrt{\frac{\alpha}{2}} \frac{\rho + \sigma}{M^4} \frac{1}{\Delta^{3/4}} \right),$$

where $\beta \equiv 4\alpha\psi^2 = 1 - \sqrt{\Delta}$. In the early universe, most energy is present in the ϕ -field, which means

$$\rho \simeq \rho_\phi = \frac{4(1 - \beta)^{3/2}}{(2\beta)^{1/2}} \psi M^4 \sinh(3\varphi/2) - \sigma. \quad (6)$$

The brane tension σ is *not* fine-tuned except in the Randall-Sundrum limit [9] $\rho \rightarrow 0$ (vacuum dominated universe) for which $\sigma = 2\psi M^4(3 - \beta)\delta$ and $\delta = 1$ [10]. The natural choice is $0 \leq \delta \leq 1$. The scalar-matter density $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$.

III. GB ASSISTED INFLATION

The inflaton equation of motion is

$$\ddot{\phi}(t) + 3H(t)\dot{\phi}(t) + V_\phi = 0, \quad (7)$$

where $V_\phi \equiv dV/d\phi$. Scalar fluctuations generated from inflation include two types of contributions, one coming from

quantum fluctuations of the ϕ -field and the other from massive Kaluza-Klein (KK) modes from the bulk. The massive scalar modes with mass $m_{\text{KK}} > 3H/2$, which are too heavy to be excited during a slow-roll inflationary era, are rapidly oscillating and their amplitudes are strongly suppressed on largest scales [11]. If the energy density of ϕ -field is very large during inflation, such as $\rho^{1/4} \gtrsim 1.5 \times 10^{16}$ GeV (as inferred by Planck and BICEP2 data), then most contributions to scalar perturbations would arise from the ϕ -field. Moreover, after a few e-folds of inflation the effect of bulk radiation energy becomes negligibly small in which case Δ (β) is constant. Under these two approximations, which are reasonably good, the amplitude of scalar (density) perturbations is given by

$$A_S^2 \equiv \frac{4}{25} \mathcal{P}_{\text{sca}}(k) = \frac{9}{25\pi^2} \frac{H^6}{V_\phi^2}. \quad (8)$$

The amplitude of primordial tensor fluctuations depends on the energy scale of inflation and nature of quantum gravitational fields that generated gravitons – the elementary particles that mediate the force of gravity. The normalized amplitude of primordial tensor perturbations was previously obtained in Refs. [12, 13], which in our notation ($\mu \equiv \psi M$), reads as

$$A_T^2 \equiv \frac{1}{25} \mathcal{P}_{\text{ten}}(k) = \frac{2}{25} \frac{\psi}{M^2 \mathcal{A}} \left(\frac{H}{2\pi} \right)^2, \quad (9)$$

$$\mathcal{A} \equiv (1 + \beta) \sqrt{1 + x^2} - (1 - \beta) x^2 \sinh^{-1} \frac{1}{x},$$

where $x \equiv H/(\psi M) = \beta^{-1/2} [(1 - \beta) \cosh \varphi - 1]^{1/2}$ is a dimensionless measure of Hubble expansion rate. The five-dimensional impact on the scalar and tensor power spectra is largely characterised through a modification of Hubble expansion rate as given in Eq. 5. The power of the scalar and tensor primordial spectra can be calculated approximately in the framework of the slow-roll approximation by evaluating the above equations at the value $\varphi = \varphi_*$ where the mode $k_* = a_* H_*$ crosses the Hubble radius for the first time. On the usual assumption that H is nearly constant throughout inflation, the amplitude of scalar density perturbations has some scale dependence due to a small variation in V_ϕ , while the tensor perturbations are roughly scale independent.

The number of e-folds of inflation $N \equiv \int H dt$ is given by

$$N \equiv \int_{\varphi_*}^{\varphi_e} H \frac{dt}{d\phi} \frac{d\phi}{dV} \frac{dV}{d\varphi} d\varphi \simeq 3 \int_{\varphi_e}^{\varphi_*} \frac{H^2}{V_\phi^2} \left(\frac{dV}{d\varphi} \right) d\varphi,$$

where the equality holds in the slow-roll approximation $\ddot{\phi} \ll 3H(t)\dot{\phi}$ and subscript “e” refers to the end of inflation. The simplest class of inflationary models is characterized by monomial potentials of the form $V(\phi) = m^{4-n}\phi^n$. For slow-roll inflation, the first two slow-roll parameters $\epsilon \equiv -\dot{H}/H^2$ and $\eta = V_{\phi\phi}/(3H^2)$ are evaluated to be

$$\epsilon = \frac{(2+n)(1-\beta)I(\varphi)}{2[N(2+n)+n]} \times \frac{\sinh \varphi [\sinh(3\varphi/2) - c]^{2-2/n}}{[(1-\beta) \cosh \varphi - 1]^2 \cosh(3\varphi/2)}, \quad (10)$$

$$\eta = \frac{3(n-1)(n+2)I(\varphi) [\sinh(3\varphi/2) - c]^{1-2/n}}{2n[N(2+n)+n] (1-\beta) \cosh \varphi - 1}, \quad (11)$$

*The universe may be slightly open at present, $(-\Omega_\kappa) \equiv \kappa/a^2 H^2 = (10^{-2} \sim 10^{-3})$, if so, $\kappa/a^2 H^2$ becomes important at low energies.

where $c \equiv (3 - \beta)\beta^{1/2}(1 - \beta)^{-3/2}\delta/\sqrt{2}$. Here we give the explicit expression of $I(\varphi)$ for $n = 2$, $n = 1$ and $n = 2/3$:

$$\begin{aligned} I(\varphi) &= \varphi - \frac{2\beta}{3} \ln(e^\varphi - 1) + (1 - \beta)(\cosh \varphi - 1) \\ &\quad + \frac{3 - \beta}{3} [\ln 3 - \ln(e^{2\varphi} + e^\varphi + 1)], \\ I(\varphi) &= \frac{(1 - \beta)}{5} \sinh \frac{5\varphi}{2} - \frac{2}{3} \sinh \frac{3\varphi}{2} + (1 - \beta) \sinh \frac{\varphi}{2}, \\ I(\varphi) &= \frac{1 - \beta}{48} (3 \cosh 4\varphi + 6 \cosh 2\varphi - 8 \cosh 3\varphi - 1). \end{aligned}$$

The scalar spectral index is given by

$$n_s - 1 \equiv \left. \frac{d \ln A_s^2}{d \ln k} \right|_{k=aH} = -6\epsilon + 2\eta. \quad (12)$$

The tensor-to-scalar ratio $r \equiv 4\mathcal{P}_{\text{ten}}/\mathcal{P}_{\text{sca}}$ is given by

$$\begin{aligned} r &= \frac{8(2 + n) I(\varphi) (1 - \beta)^{3/2} |2\beta|^{1/2}}{N(2 + n) + n} \frac{\mathcal{A}}{\mathcal{A}} \\ &\quad \times \frac{[\sinh(3\varphi/2) - c]^{2-2/n}}{[(1 - \beta) \cosh \varphi - 1]^2}. \end{aligned} \quad (13)$$

The above results are valid for $0 < |\beta| \ll 1$ and $V(\phi) \gg \sigma$ and they improve the expressions given in [14] where the limit $\beta \rightarrow 0$ was taken but erratically. The results corresponding to an exponential inflation $V(\phi) \propto \exp[\gamma\phi/M_P]$ [15] are obtained by taking $n \rightarrow \infty$. Inflation begins at $\varphi = \varphi_*$ and depending on the form of the potentials we observe that $\varphi_* \sim (0.6 - 2.5)$; inflation begins at a larger field value for an exponential potential. As shown in Fig. 1, inflation has a natural exit ($\epsilon > 1$) only if $\beta > 0$ (or $\lambda < 0$ for $\alpha > 0$).

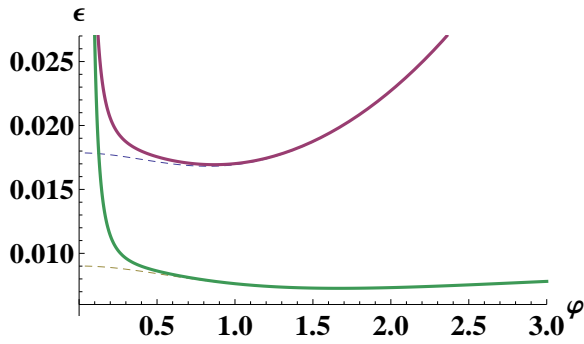


FIG. 1: ϵ vs φ : $V \propto \exp[\gamma\phi/m_P]$ (upper plot) and $V \propto \phi^2$ (lower plot) with $N_* = 55$, $\beta = 0$ (dotted) and $\beta = 10^{-3}$ (solid) lines.

In Fig. 2 color bands correspond to $\beta > 0$ and the bold lines correspond to $\beta \approx 0$. The effect of β on monomial potentials $V \propto \phi^n$ is *not* uniform for all values of n . Especially, for $m^2\phi^2$ inflation, a positive β suppresses the amplitude of primordial tensor (gravitational waves) fluctuations more than the scalar (density) primordial fluctuations. This effect is opposite for other values of n , including $n = 1$ and $n = 2/3$. In all cases the value of r is greater than their values in general relativity (GR) [1]. Note that the limit $\beta \rightarrow 0$ corresponds to Randall-Sundrum braneworld cosmology [11], *not* to the GR limit. For $m^2\phi^2$ inflation in GR, $n_s \sim 0.96$ corresponds to $r \sim 0.157$ (see e.g. [16] or the single solid line in Figs. 2 and 3). For $\beta \approx 0$, $n_s = 0.96$ implies $r = 0.1742$

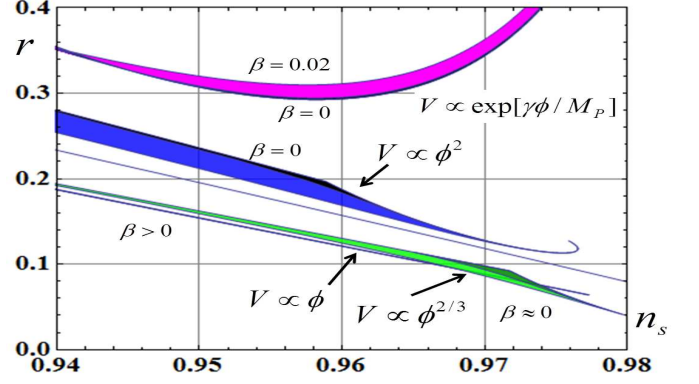


FIG. 2: The GB-assisted ϕ^n -inflation: tensor-to-scalar ratio vs scalar spectral index with $N_* = 60$, $0 < \beta \leq 0.02$ and $\delta = 0$.

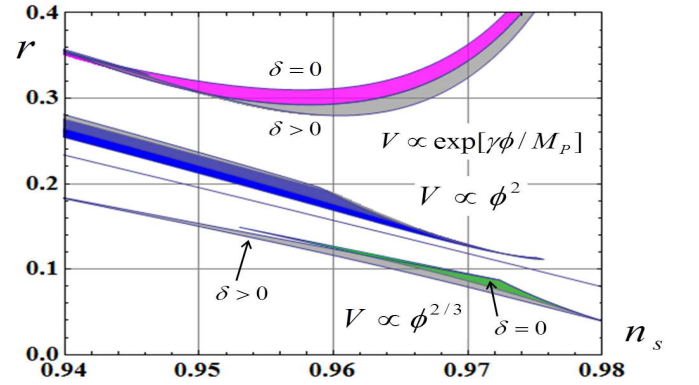


FIG. 3: As in Fig. 2 but $0 \leq \delta < 0.5$.

(0.1885) for $N_* = 50$ (60). The value of r decreases once β is increased; for example, for $\beta \simeq 10^{-4}$, $n_s \simeq 0.96$ implies $r \simeq 0.1741$ (0.1822) for $N_* = 50$ (60). These values are close to the central value of r reported by BICEP2 collaboration. For $V \propto \phi^{2/3}$ -inflation, the value of n_s (r) is relatively large (small). For $\beta \approx 0$, $n_s \sim 0.971 - 0.980$ and $r \sim 0.04 - 0.1$, which both are away from the mean values of n_s and r reported by Planck and BICEP2 collaborations. If $\beta > 0$, as shown in Fig. 2, a smaller (larger) value of n_s (r) can also be obtained; viz, $(n_s, r) = (0.96, 0.13)$. This kind of suppression in scalar power with a larger tensor-to-scalar ratio at higher energies (in Gauss-Bonnet regime) can help to reconcile the Planck and BICEP2 data in a single framework.

For a small coupling constant like $\beta < 0.001$, we get $c < 0.067$; the effect of brane tension is small if the energy scale of inflation is large. In Fig. 3, color bands correspond to $\delta = 0$ and the grey bands to $\delta = 1/2$. A positive δ lowers the value of r for an exponential potential, while it increases r for $m^2\phi^2$ inflation. For $V(\phi) \propto \phi$, the values of r and n_s do not depend on σ (or δ) and they depend on N_* modestly; for $n_s \simeq 0.96$, $r \simeq 0.119 - 0.121$ with $N_* \sim 50 - 60$.

In order to constrain the model parameters we use the COBE normalization for amplitude of scalar perturbations used by Planck collaboration, $A_* \simeq V^3/(12\pi^2 M_P^6 V_\phi^2) \simeq 22 \times 10^{-10}$. As shown in Fig. 4, when $n_s \simeq 0.96$, we get

$$A_* \simeq 15 \times (M/M_P)^6 \rightarrow M \simeq 5.51 \times 10^{16} \text{ GeV}.$$

Further, β and ψ may be constrained by using the dimensional reduction relation $\psi m_P^2 = (1 + \beta)M^2$ [6] between the four-

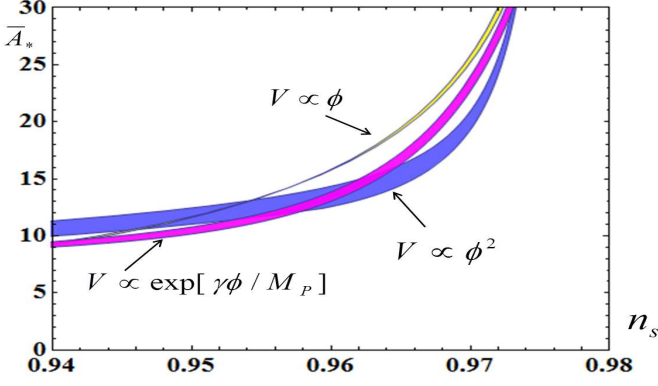


FIG. 4: The COBE normalized amplitude of scalar perturbations $\bar{A}_* \equiv (M_P/M)^6 \times A_*$ vs n_s with $N_* = 60$ and $0.02 > \beta > 0$.

and five-dimensional Planck masses. For GB assisted $m^2\phi^2$ inflation, with $N_* \simeq 55$ and $n_s \simeq 0.96$, we observe that

$$12 < \xi^2 < 48, \quad \xi^2 \equiv \frac{4 \times 10^4 \cdot \alpha \cdot m^2}{M^2}. \quad (14)$$

A deviation from $n_s \simeq 0.96$ changes this bound slightly; for example, if $n_s \simeq 0.963$, then $25 < \xi^2 < 60$. During inflation $m \gtrsim H \simeq 1 \times 10^{14}$ GeV, which means $\alpha \sim 91 - 364$ and $\beta \sim (1 - 4) \times 10^{-4}$. For a small-mass inflaton field, $m \simeq 2 \times 10^{13}$ GeV, the bound on β is tighter, $\beta \sim (25 - 130) \times 10^{-4}$. These numbers are compatible with current observations related to inflationary era for a wide range of the energy scale of inflation and the number of e-folds. During the early phase of inflation, $V = m^2\phi_*^2 \simeq (1.5 \times 10^{16} \text{ GeV})^4$, which implies that $\phi_* \simeq 0.89 \times M_P$; we have a stage of inflation at $\phi_* \lesssim M_P$. Inflation ends at $\varphi = \varphi_e \simeq 0.01$, or when $V_{\text{end}}^{1/4} \simeq 0.16 \times M = 8.9 \times 10^{15}$ GeV and $\phi_{\text{end}} \simeq 0.33 M_P$. It follows that $\Delta\phi \equiv \phi_* - \phi_{\text{end}}$, the change in ϕ after the scale k_* leaves the horizon, $\Delta\phi \simeq 0.56 M_P$. This estimates agrees with Lyth's recent discussion in [17] and there is no Super-Planckian excursion of the inflaton field. In fact, the Lyth bound $\Delta\phi \gtrsim M_P \sqrt{r/4\pi}$ may not apply to Gauss-Bonnet assisted inflation since the value of r varies with both the energy scale of inflation φ_* and the number of e-folds.

IV. DISCUSSION AND SOME COMMENTS ON THE LITERATURE

Though Ref. [8] obtained the set of 4D equations using a covariant formalism, their equations were not written in a form from which one can easily obtain the expression of Hubble squared parameter (except in the $\alpha \rightarrow 0$ limit). There are no fundamental disagreements with results in Refs. [7, 8], which are perhaps correct. The set of equations given in [8] are in an abstract form, which are less useful at least for studying inflationary solutions since the relation between the Hubble-squared parameter and the scalar-matter density or the dimensionless scale φ related to the scale of inflation was not established. We have presented results in terms of the model parameters like ψ , β and M as they can be linked to inflationary variables.

The plots in Ref. [14] are not quite right (except the first one) since $\beta \rightarrow 0$ limit was taken in most of their discussions

after Eq. (18). In this limit, one would be studying Randall-Sundrum type braneworld inflation; in fact, $\beta = 0$ solutions do not characterize the full effect of R^2 terms, since the effects of R^2 corrections cannot be accommodated just by letting φ run. One more drawback in the analysis of Ref. [14] is that the authors used the RS type tuning for brane-tension, $\sigma = 2\psi M^4(3 - \beta)$, which only holds in the limit $\rho_\phi \rightarrow 0$, $H \rightarrow 0$ and $\mathcal{E} \rightarrow 0$ but not if any of these quantities is not zero. It is important to realize, in the present model, that inflation ends only when β is positive, but not when $\beta = 0$. This is an important difference from Ref. [14]. More importantly, we have got a success to constrain the model parameters like M , β , and/or $\lambda\alpha$ for the first time by using inflationary constraints, such as, the COBE normalized amplitude of scalar fluctuation and spectral indices. This result is truly remarkable.

The major outcomes of this paper are the results given in Eqs.(10)-(14) and the plots in Figs. 1 to 4. For a completeness, we also expressed the 4D field equations in a form most appropriate, which were known before in one or another form.

We have also shown that a steep inflation may be compatible with the BICEP2 results (within 2σ CL, $r < 0.27$) provided that the number of e-folds of inflation $N \gtrsim 70$ and the brane tension is also large. This improves the earlier discussions in Ref. [15]. In this last reference, some constraints on n_s were derived for an exponential potential for different values of N . Constraints on r that are compatible with constraints on n_s were not considered there – a parametric plot between n_s and r would help us to compare and confront theoretical results with Planck and BICEP2 constraints for inflationary parameters as discussed above.

Here we make one more comment. As long as the GB coupling is nonzero (no matter how tiny), one would never go to RS regime because the RS regime means $\alpha = 0$ absolutely. It is not true that inflation begins in the GB regime and ends in the RS regime; the drop in energy scale $V^{1/4}$ during inflation is generically only an order of magnitude difference, which means inflation can occur solely during a phase where $H^2 \propto (\rho + \sigma)^{2/3}$ [12]. At a later epoch H^2 scales as $(\rho + \sigma)^2$ and this scaling relation can be seen both with $\alpha = 0$ and $\alpha \neq 0$. The \mathcal{R}^2 -type corrections (of a Gauss-Bonnet form) would lead to graceful exit from inflation for a number of scalar potentials, provided that $\beta > 0$. Moreover, these corrections are important at the earliest epoch, though their contributions diminish rapidly after inflation (more precisely, after reheating) all the way to the epochs of baryogenesis, nucleosynthesis and at the present epoch.

V. CONCLUSION

In this paper we have identified a gravitational theory where inflation has a natural exit. For $V \propto \phi^n$ -type inflation with $n \geq 1$, it is shown that the Gauss-Bonnet term \mathcal{R}^2 can generate a suppression in scalar power at large scales along with reasonable amplitudes of primordial scalar and tensor perturbations ($r \sim 0.12 - 0.20$, $n_s \simeq 0.96$). If BICEP2 is going to confirm their reported result that $r = 0.19^{+0.07}_{-0.05}$ [3] then an exponential inflation can be compatible with the result only if the brane tension or the bare cosmological constant is nonzero and the number of e-folds of the cosmic inflation is significantly large, $N_* \gtrsim 70$. The GB-assisted $m^2\phi^2$ inflation is in agreement with BICEP2 for a wide range of the energy

scale of inflation and number of e-folds $47 < N_* < 65$. The $m^2\phi^2$ inflation fits better with the BICEP2 result as compared to other forms of scalar potentials, such as, $V \propto \phi^{2/3}$ and $V \propto \phi$. A GB-assisted natural inflation model characterized by the potential $V = V_0 (1 \pm \cos(n\phi/M_P))$ which approximates the $m^2\phi^2$ potential for $n \ll 1$ is also compatible with BICEP2 [18].

The gravitational theory discussed in this paper offer simplest and at the same time important mechanism to generate a larger tensor-to-scalar ratio (as compared to the results in GR) that can consistently address the apparent discrepancy between the Planck upper bound and the BICEP2 detection of $r = 0.16^{+0.06}_{-0.05}$ (after subtracting an estimated background) because the GB-assisted inflation leads to a scenario where tensor perturbations are roughly scale independent, while some scale dependence of scalar density perturbations could relax the Planck constraint and bring the two results into agreement.

The GB coupling constant α is tightly constrained in combination along with another constant λ associated with the curvature of the five-dimensional spacetime, namely $(-\lambda\alpha) \simeq (3 - 300) \times 10^{-5}$, making a prediction that could be confirmed or falsified by a future detection of some scale dependence of scalar density perturbations and hence the measurement of a small but nontrivial running of the scalar spectral index and possibly a small non-Gaussianity parameter. This paper makes a major contribution in the field of inflation-

ary cosmology by limiting coupling parameters of Einstein-Gauss-Bonnet gravity that are compatible with BICEP2 and Planck constraints for primordial cosmic inflation.

The model investigated here is a natural generalization of RS model (also called 5-dimensional warped geometry theory). In string theory, like in type II theory, one adds a five sphere S^5 to get a ten-dimensional space. The five-sphere is usually related to the scalars, fermions and anti-symmetric form fields in the super-symmetric Yang-Mills theory. For other theories the sphere is replaced by other manifolds, or it might even not be there. It is logical to assume that the effect of scalar(s) is encoded in the four-dimensional effective scalar Lagrangian as implicitly assumed in the paper.

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